PERTURBATIVE QUANTUM FIELD THEORY[a]

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This talk introduces perturbative quantum field on a heuristic level. It is directed at an audience familiar with elements of quantum mechanics, but not necessarily with high energy physics. It includes a discussion of the strategies behind experimental tests of fundamental theories, and of the field theory interpretations of these tests.

Prologue: the Lunar Apsides

Two and a half centuries ago, within fifty years of its author's death, Newtonian physics was the leading description of celestial and terrestrial motion. Newtonian gravitation, however, was not universally accepted, even among the great *savants* whose names are still familiar today. It worked as an approximation, that much was clear, and the beauty of the Newtonian solution to the two-body problem was indisputable. Some perceived the theory, and its elegant inverse-square law, as well-nigh perfect, but others felt that the law, and its action-at-a-distance, could not be the final word. Thus, they set out to delineate its limitations. To these thinkers, Newtonian gravitation was at best a convenient accounting for appearances.

Small but cumulative changes in planetary orbits were ideal challenges for Newtonian gravity. Here were what we now call precision tests of the theory, much more demanding than the oversimplified two-body problem. Wonderfully accurate data on lunar motion (studied in part for its application to the rapidly developing science of navigation) seemed a natural place to start

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the search. The correspondence on this issue of the great mathematician and physicist Leonhard Euler, and of two of his scarcely lesser contemporaries, Jean d'Alembert and Alexis Clairaut, illustrate a pace of intellectual advance that we may well envy today, as well as recognizably human generosity and jealousy, and the eternal desire for priority. [1]

Between the fall of 1747 and the spring of 1749, the motion of the lunar apsides (apogee and perigee) was a burning question for the Royal Academies of Paris, Berlin and Saint Petersburg. Four milestones in the development of its theory are illustrated by these communications:

- Sept. 1747 (Euler to Clairaut): "...the forces that act on the moon do not follow the rule of Newton."

- Nov. 1747 (Clairaut to French Royal Academy): [Newton's theory of gravitation false]

- June 1748 (d'Alembert to the Swiss mathematician Cramer): "...the gravitation of the moon to the sun will not explain [the] irregularities of its motion."

- May 1749 (Clairaut to French Royal Academy): [Terms previously neglected show Newton's law was right all along.]

At various stages in the discussion, the possibility of a $1/r^4$ correction to earth-moon gravity, perhaps associated with magnetism, was considered by these authors. Eventually, however, the theory of lunar motion developed into the theory of the three-body problem, and it was here, in the influence of the sun, that an explanation adequate to the observations was found. This was the beginning of perturbation theory, developed by Lagrange and Laplace, on through to Poincaré and into the present day.

Euler never accepted Newtonian gravity, even as he went on to perfect a Newtonian lunar theory, building on Clairaut's discovery. He even arranged that Clairaut be awarded a prize by the Russian Royal Academy for his work. On the other hand, relations between d'Alembert and Clairaut never recovered from their competition on the lunar apsides.

1 Introduction

In our study of fundamental forces and constituents of matter, we find ourselves following the tradition recalled above. We are heirs to the historic discovery of the quantum field theories that make up the standard model. We are learning to spin out the consequences of these theories, while searching for a horizon

beyond which they fail, and must be replaced by a new picture of fundamental science.

In this talk, I will describe the role of perturbative quantum field theory in our understanding of fundamental processes in nature, beginning from basic concepts of quantum mechanics. We should keep in mind that any such discussion is in the context of what I'd like to call "postmodern" particle physics, with its dialectic of paired concepts, such as *standard* versus *new*, *effective* versus *fundamental* and *weak coupling* versus *strong coupling*. Although usually employed in the search for new physics, these ideas illuminate, and are illuminated by, quantum chromodynamics (QCD), the theory of the strong interactions in the standard model. Even while we are engaged in the search for physics beyond the standard model, we are in a veritable golden age of data on the strong interactions, in terms of coverage and quality. The challenge of understanding this constellation of data is, in some ways, similar to that which confronted Euler and his contemporaries so long ago.

The perturbative approach begins with a "solvable" system, a classical or quantum mechanical Hamiltonian, H, whose time-development can be quantified fully. Perturbation theory organizes the effects of a small modification of H, the perturbation. In celestial mechanics, this may be the influence of a far-off "third body", in field theory, a small amplitude for the quantum-mechanical production of a new particle. At any given time, the perturbation is supposed to be a small contribution to the energy or its expectation value; over long periods, its influence may be profound.

I will begin with a quick introduction to perturbative quantum field theory, and go on to discuss how perturbative techniques are used to search for new physical phenomena. This will lead us to the technique of separating scales, applied to ultraviolet divergences and renormalization, on the one hand, and infrared divergences and infrared safety on the other. I will end with a brief discussion of how factorization extends the applicability of quantum field theory in the modern experimental program.

General-interest and technical discussions of the standard model, and of the high energy experimental program, may be found through the web sites of the major experimental laboratories, including CERN (www.cern.ch), DESY (www.desy.de), Fermilab (www.fnal.gov), and SLAC (www.slac.stanford.edu), and of the Particle Data Group (pdg.lbl.gov). A review of recent (and rapidly-evolving) trends in physics beyond the standard model is in Ref. [2]. These considerations serve as the leading motivation for the endeavor of high energy physics. The quantum-mechanical basis of the high energy tests that they suggest is one focus of this talk. More technical introductions to another primary focus, the standard model example of perturbative QCD, are in Refs.

[3], [4] and [5].

2 Perturbative Quantum Field Theory

2.1 Lagrangians and Fields

The quantization of a field theory, particularly a gauge theory like electrodynamics, is rife with subtlety. Yet, the underlying principles are straightforward enough, and follow the general rules of quantum mechanics. Let's ignore the fine points, and follow a familiar path, starting with a classical Lagrangian for electrodynamics, and its much younger sibling, chromodynamics. The group theory underlying the latter need not concern us too much. For now, we observe that the Lagrangian for electrodynamics may be represented as

$$\mathrm{L_{EM}} = K_{\mathrm{electron}} - \int d^3x \left\{ e\, J_{\mathrm{electron}} \cdot A_{\mathrm{EM}} - \frac{1}{8\pi} \left(E^2 - B^2 \right)_{\mathrm{EM}} \right\}, \quad (1)$$

where K_{electron} stands for the kinetic energy of the electron, and where the following two terms represent the interaction potential energy due to the electron current J and electromagnetic field A_{EM}, and the Lagrangian for the free electromagnetic field, in terms of the electric and magnetic fields, themselves determined by derivatives of A_{EM}. In one or another form, this expression can be found in textbooks for classical electromagnetism.

In these broad terms, the classical Lagrangian for chromodynamics looks much the same,

$$\mathrm{L_{QCD}} = K_{\mathrm{quark}} - \int d^3x \left\{ g\, J_{\mathrm{quark}} \cdot A_{\mathrm{QCD}} - \frac{1}{8\pi} \left(E^2 - B^2 \right)_{\mathrm{QCD}} \right\}, \quad (2)$$

as the sum of quark kinetic, quark-chromodynamic field interaction, and pure chromodynamic field terms. In a slightly more elaborate but still schematic form, we may represent $\mathrm{L_{QCD}}$ in terms of the fields themselves:

$$\mathrm{L_{QCD}} = \int d^3x \left\{ \sum_q \bar{q}(x)\, \partial\, q(x) \right\}$$
$$-g \int d^3x \left\{ \sum_q \bar{q}(x)\, q(x)\, A_{\mathrm{QCD}} \right\}$$
$$+ \int d^3x \left\{ (\partial A_{\mathrm{QCD}})^2 - g(\partial A_{\mathrm{QCD}})\, A_{\mathrm{QCD}}^2 - g^2 A_{\mathrm{QCD}}^4 \right\}, \quad (3)$$

where q, $\bar{q}$ represent spinor fields (quarks), and A_{QCD} vector fields (gluons) and ∂ represents space-time derivatives. When fully expanded, $\mathrm{L_{QED}}$ looks

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much the same in terms of electron and photon fields, but without the cubic and quartic terms in A.

Even in this simplified form, Eq. (3) illustrates the basic principle at the heart of the standard model, gauge symmetry. We demand the unobservability of the "gauge" transformation $q'(x) = \exp[i\alpha(x)]q(x)$, even for a position-dependent (local) phase change $\alpha(x)$. For QCD, the spinor field $q(x)$ is a vector in an internal, color space, and $\alpha(x)$ is a matrix. Any such transformation manifestly changes the first, kinetic term in (3), as soon as the phase is space-time dependent. In a gauge theory, however, this change is cancelled by a corresponding change in the interaction, $J \cdot A$, term, when the vector field is modified in a corresponding manner: $A'_\mu(x) = \exp[i\alpha(x)]A_\mu(x)\exp[-i\alpha(x)] + (i/g)(\partial_\mu \exp[i\alpha(x)])\exp[-i\alpha(x)]$. When the action for the vector field is built along the lines of the final terms in Eq. (3), the Lagrangian as a whole can be made invariant under the combined transformations. This binding-up of the symmetry properties of spinor and vector fields is encoded into every aspect of the standard model.

The next step on the road to quantization is to transform the Lagrangian into a Hamiltonian,

$$L \rightarrow H = H^{(0)} + gV_1 + g^2 V_2 \,, \tag{4}$$

where $H^{(0)}$ is quadratic in the fields. In the perturbative approach, we begin by "solving" $H^{(0)}$ that is, by identifying its eigenstates; they will be free electrons and photons in QED, and free quarks and gluons in QCD. As we shall see, each term in $V = gV_1 + g^2 V_2$ defines an elementary process that mixes free states. Perturbation theory computes mixing as a power series in g, an approximation to the true states and processes of the theory.

2.2 From $H^{(0)}$ to Free-Particle States

Let's construct the energy eigenstates of $H^{(0)}$. The natural degrees of freedom are the spatial Fourier transforms of the fields themselves, time-dependent functions characterized by wave numbers $\vec{k}$:

$$q(x), \; A^\mu(x) \rightarrow \tilde{q}(\vec{k}, x^0), \; \tilde{A}^\mu(\vec{k}, x^0) \,, \tag{5}$$

with $x^0 \equiv ct$ a measure of time in units of length. We use $H^{(0)}$ as a starting point in the perturbative analysis of the full Hamiltonian, $H = H^{(0)}(q, A_\mu) + V(q, A_\mu)$. The analysis is quite general, depending only in the details. By construction, $H^{(0)}$ is *quadratic* in each of the degrees of freedom in (5). In the classical free theory, this means that the fields obey a linear equation, which implies that solutions obey a principle of superposition: the sum of two

solutions is also a solution. In the wave number space of Eq. (5), the system always simplifies to an independent harmonic oscillator equation of motion for each $\vec{k}$, with solutions

$$\tilde{q}(\vec{k}, x^0) = b(\vec{k})e^{-i\omega(\vec{k})\,t}\,, \tag{6}$$

where $\omega(\vec{k})$ is the frequency associated with wave vector $\vec{k}$, and $b(\vec{k})$ is the amplitude of the corresponding wave. Superposition implies that each $b(\vec{k})$ is independent.

The quantization of such plane waves is particularly simple: each amplitude $b(\vec{k})$ is quantized in the manner of a harmonic oscillator. The quantum system resides in states characterized by quantized $|b(\vec{k})|^2$ for each $\vec{k}$. These "free" states may be written as

$$|m> = |\{\vec{k}_i\}, \{\vec{q}_j\} >\,, \tag{7}$$

where each $\vec{k}_i$ denotes a quantum excitation of wave vector $\vec{k}_i$ for the spinor field, and $\vec{q}_j$ for the vector field. The energies of the states (neglecting zero-point energies) are

$$H^{(0)}|m> = \hbar \left(\sum_i \omega_i(\vec{k}) + \sum_j \omega_j(\vec{q}) \right) |m> \equiv S_m|m>\,, \tag{8}$$

with

$$\omega(\vec{k}) = \sqrt{\vec{k}^2 c^2 + M^2 c^4/\hbar^2} = \frac{1}{\hbar}\sqrt{\vec{p}^2 c^2 + M^2 c^4}$$

$$\omega(\vec{q}) = |\vec{q}| = \frac{1}{\hbar} E(\vec{q})\,, \tag{9}$$

where M is the mass of the spinor particle. The second relation in Eq. (8) defines S_m. The states $|m>$, being exact eigenstates of the Hamiltonian, are *stationary*, that is, the wave numbers describe the states for all time. Another way of saying this is that there is no scattering. This is the quantum version of the principle of superposition for the equation of motion of the classical field theory. A typical state is illustrated in Fig. 1a, in which the horizontal direction represents time, and two waves, the straight one representing an "electron" and the curved one a "photon", pass through each other without interacting.

2.3   The Interaction Mixes the Free States

Scattering, interpreted as a change in the wave numbers of excitations in the system, is associated with the potential terms V in the interacting Hamiltonian

of Eq. (4). To see why, we only need to solve the Schrödinger equation in the interacting theory:

$$i\hbar \frac{\partial}{\partial t}|\psi(t)> = \left(H^{(0)} + V\right)|\psi(t)>, \tag{10}$$

with a free-state boundary condition,

$$|\psi(-\infty)> = |m_0> . \tag{11}$$

The physical content of the boundary condition is intuitively clear; it corresponds to the preparation of an experiment in which isolated particles are arranged to collide. This is what any accelerator does.

In the perturbative solution to the interacting Schrödinger equation, we assume that solutions to the free equation are complete,

$$\sum_m |m><m| = 1. \tag{12}$$

Introducing the notation,

$$V_{j,i} \equiv \langle m_j|V|m_i\rangle, \tag{13}$$

we may expand a general solution $|\psi(t)>$ in terms of the states of the free ($V = 0$) theory,

$$|\psi(t)> = \sum_m |m><m|\psi(t)> . \tag{14}$$

We readily verify that the following infinite expansion for the matrix elements in (14) constitutes a solution to Eq. (10),

$$<m_n|\psi(t)> = \sum_{n=0}^{\infty} \sum_{m_1...m_n} e^{-iS_n t/\hbar} \, (-i/\hbar)^n \, V_{n,n-1} \, V_{n-1,n-2} \times \ldots \times V_{1,0}$$

$$\times \int_{-\infty}^t d\tau_n \, e^{-i(S_{n-1}-S_n)\tau_n/\hbar} \int_{-\infty}^{\tau_n} d\tau_{n-1} e^{-i(S_{n-2}-S_{n-1})\tau_{n-1}/\hbar}$$

$$\times \ldots \times \int_{-\infty}^{\tau_2} d\tau_1 e^{-i(S_0-S_1)\tau_1/\hbar}, \tag{15}$$

where the phases S_m are defined by the free theory, through Eq. (8) above.

Although the expression for the matrix elements in Eq. (15) may seem a little complicated, it has a nice interpretation. Each term in the sums $\sum_{m_1...m_n}$ defines the evolution of the system, from an initial free state $|m_0>$ at $t =$

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$-\infty$ to the observed state $|m_n>$ at time t, through a sequence of free states $|m_i>$, in which the system resides for a time $\tau_{i+1} - \tau_i$. At each step, the transition between states is governed by a matrix element in the free theory, V_{m_{i+1},m_i}. The intermediate states need not be equal in energy to the initial state, as reflected in the phases that oscillate according to energy differences and intermediate times. On the other hand, if the potential is translation invariant, the V's will conserve momentum, by being proportional to delta functions of the form $\delta^3(\vec{p}_{m_{i+1}} - \vec{p}_{m_i})$.

An example is shown in Fig. 1b, in which an "electron-photon" system, with initial state m_0, passes through two intermediate states, m_1 and m_2, ending up in a state, m_3, with an electron and two photons. Each vertex in this "time-ordered diagram" represents one of the matrix elements $V_{j,i}$. The lines meeting at any vertex match the fields in the corresponding term in the potential. Thus, for QED, with potential $e\bar{q}qA_{\mathrm{QED}}$, each vertex connects two spinor and one vector line. QCD has, in addition, three-vector (gA^3_{QCD}) and four-vector ($g^2A^4_{\mathrm{QCD}}$) vertices.

2.4   Time-ordered Perturbation Theory

To put Eq. (15) into a more standard form, we simply do the time integrals, which are trivial. For every τ_i, $i < n$, they give $i/(S_{m_i} - S_0)$. If we take the limit, $t \to \infty$ for the last time, τ_n, its integral gives $2\pi\delta(S_n - S_0)$, which enforces energy conservation over long enough times.

The result is time-ordered (sometimes called "old-fashioned") perturbation theory. [c] Let us define $\lim_{t\to\infty} < m|\psi(t) > \equiv \Gamma(p)$, where p denotes collectively the wave numbers of particles in the initial state m_0 and the final state m_n. The general form for Γ, suppressing the energy conservation delta function, is [d]

$$\Gamma(p) = \prod_{\mathrm{loops}\ i} \int \frac{d^3\ell_i}{(2\pi)^3} \prod_{\mathrm{lines}\ j} \frac{1}{2\omega_j(k_j)} \prod_{\mathrm{states}\ a} \frac{1}{E_a - S_a + i\epsilon} N(p, \ell_i). \qquad (16)$$

The function N collects overall factors and polynomials in momenta from the product of the matrix elements V. These depend on the spin and other quan-

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[c] From here on, we shall go over to units in which all momenta and masses are specified in dimensions of (length)$^{-1}$. Thus, for an energy E or mass M, we define $p_0 = E/\hbar c$, and $m = Mc/\hbar$, respectively. We shall also drop the arrow over wave numbers.

[d] The term $i\epsilon$ in the denominators defines the integrals at $t = -\infty$ in Eq. (15) by attributing a small imaginary part to the incoming energy S_0.
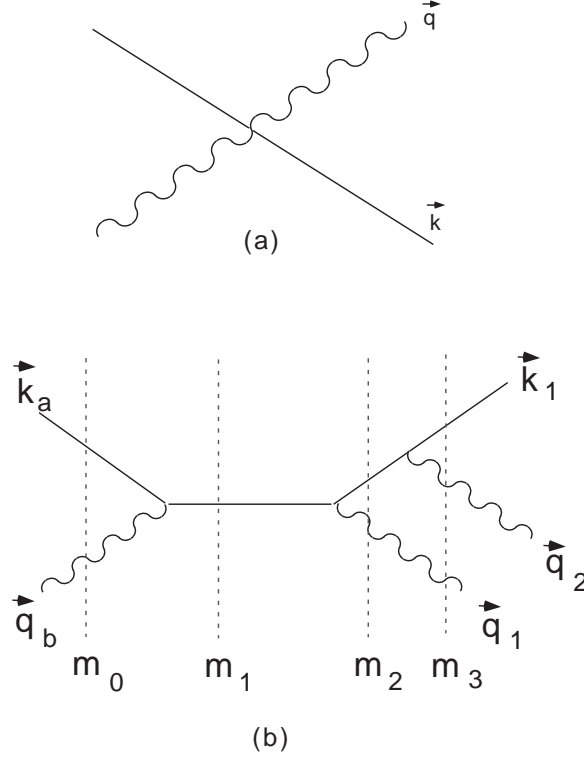
Figure 1: (a) Representation of a state in the free theory. (b) Mixing of states in the interacting theory.

tum numbers of the fields. The sum over states in Eq. (16) is

$$\sum_{\text{states}} = \prod_{\text{loops } i} \int \frac{d^3 \ell_i}{(2\pi)^3} \prod_{\text{lines } j} \frac{1}{2\omega_j(k_j)} .$$ (17)

Here the independent, or "loop" momenta, ℓ_i are the momenta left over after all momentum conservation delta functions in the matrix elements have been employed. An example is shown in Fig. 2, which is a "self-energy", the amplitude for a particle, of initial energy p_0, to evolve into itself, at second order in a potential that links three fields together. This amplitutde is

$$\Gamma_1 + \Gamma_2 =$$ (18)

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$$\sum_{2-\text{particle states}} \left(\frac{1}{p_0 - \omega(k_1) - \omega(k_2) + i\epsilon} + \frac{1}{-p_0 - \omega(k_1) - \omega(k_2) + i\epsilon} \right),$$

where we have suppressed all overall factors and delta functions. The two terms correspond to the two time orderings. In the first of the orderings, the potential transforms the particle into a two-particle state, whose energy is $\omega(k_1) + \omega(k_2)$. In the second, the potential actually creates three particles out of the vacuum, so that the intermediate state contains four particles, and has energy $2p_0 + \omega(k_1) + \omega(k_2)$, as reflected in the different "energy denominator" in this case.

Because energies are not Lorentz invariant, the contributions of individual time-orderings to an amplitude are also not invariant. Nevertheless, their sum is. This overall invariance is made manifest by the formalism of Feynman diagrams, each of which summarizes the complete set of time orderings of the same topology. Thus, the two time-ordered diagrams of Fig. 2 may be represented by a single invariant diagram, without relative ordering of its vertices. The price for doing this is to introduce a new energy integral for each loop, and to replace energy denominators by covariant propagators, one for each line. For the self-energy above, we find

$$G(p) \equiv \Gamma_1 + \Gamma_2 = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 - m^2 + i\epsilon} \frac{1}{(p-k)^2 - m^2 + i\epsilon}, \qquad (19)$$

or more generally, summing over all orders in an arbitrary diagram,

$$G(p) \equiv \sum_{t-\text{orders}} \Gamma(p) = \prod_{\text{loops } i} \int \frac{d^4 \ell_i}{(2\pi)^4} \prod_{\text{lines } j} \frac{i}{k_j^2(p, \ell_i) - m_j^2 + i\epsilon} \tilde{N}(p, \ell_i). \quad (20)$$

The factors $\tilde{N}$ are again associated with the quantum number content of the fields.

This brings our development of the formalism of quantum field theory to a conclusion. Luckily, to understand the basic issues in the phenomenology of quantum field theory, it is not necessary to delve into too many of the theory-dependent details. Most of our discussion will rely on the description of scattering amplitudes as sums over all possible paths through intermediate states.

3 Using Perturbation Theory: Searches

It is not too misleading to characterize the aim of high energy physics as the identification of an underlying Hamiltonian, or action, that governs the time
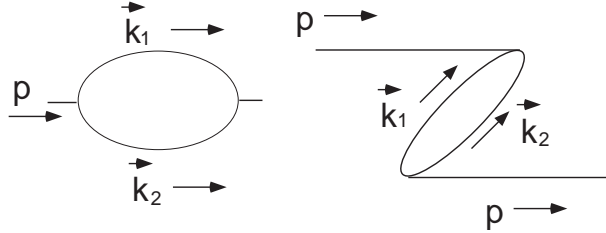
Figure 2: Time ordered diagrams for a self-energy.

development of the universe. We think that we know parts of that Hamiltonian, at least its effective form at the energy scales that we can investigate directly. In experiments, we try to learn about the parts that we do not yet know. The essential strategy is simple: predict the outcomes of scattering experiments based upon the known Hamiltonian, and look for deviations. As deviations are found, identify a modified Hamiltonian that accounts for them, and which can be tested through its predictions. Of course, the experiments that we choose to test our current theory will depend on how we perceive its successes and its shortcomings.

3.1   The Cycle of Tests

The cycle of testing to which we have just referred proceeds through the computation of quantum-mechanical scattering amplitudes, and hence probabilities. A broad summary is as follows.

- Lists of states available to us are the eigenstates of the contemporary $H^{(0)}$.

- The rules by which states mix are given by the contemporary V.

- It is useful to classify states according to symmetry generators S_i (momenta and charges) that commute with the contemporary $H = H^{(0)} + V$. States are identified by their conserved charges, as well as their momenta: $S_i \left| m, \{s_i\} \right\rangle = s_i \left| m, \{s_i\} \right\rangle$.

In this context, *new physics* may require us to make changes.

- We may have to add new states to our list, and hence reformulate $H^{(0)}$. Old states may turn out to be composites of new states, and $H^{(0)}$ may well simplify in the process.

- We may have to add new rules for changing states, and hence modify the potential, perhaps adding a new term, δV.

- If $[\delta V, S_i] \neq 0$, some symmetry S_i of our original Hamiltonian is violated, which implies that the corresponding charge s_i may not be conserved. On the other hand, new states and interactions may possess new symmetries and conserved quantum numbers.

3.2   Favorite Tests

Practitioners of high energy physics have developed, over time, a toolbox of generic experiments, sensitive to the capabilities of quantum mechanical scattering.

- As the energy increases, we are able to produce states of higher mass. When the energy is just right for the production of a particle through a hitherto unseen term in the Lagrangian by a combination of known particles, a state with the new particle is formed. This matching of energies expresses itself through an increase in the amplitude, and hence the probability, as the energy deficit decreases. An example is shown in Fig. 3, which is the cross section (the probability normalized to the density of colliding particles), for the production of particle-antiparticle pairs of leptons through the annihilation of pairs of quarks. Generally, the cross section decreases with energy, but when the energy of the quark-antiquark pair matches the mass of the Z boson, at around 90 GeV, the cross section increases dramatically. This data was taken by the CDF collaboration to scan for new physics at yet higher energies. [6] As we can see, there is no sign yet.

  The LEP II accelerator at CERN has searched for the Higgs scalar particle, H, through the reaction $e^+ + e^- \rightarrow \text{Z} + \text{H}$. The Higgs is the only remaining state, and field, that is part of the standard model Hamiltonian, but which has not yet been observed definitively. In the standard model, we can readily compute the scattering amplitude for this process as a function of the mass of the Higgs scalar. If the Higgs particle is found in this way, its mass – a previously unknown parameter of the Hamiltonian – will be determined, and the Hamiltonian of the standard model dramatically verified. Other popular searches are for particles whose presence is implied by supersymmetry, or by "extra" space dimensions.

- We may infer the presence of very heavy states even when we cannot produce them directly, if they couple to known fields through terms in

the Hamiltonian. Very heavy particles, which appear in virtual states that live for only a very short time may produce nonstandard *contact terms*, through which standard fields appear to interact. Thus, in high energy experiments, it is important to look for new interactions between leptons and quarks, or quarks and quarks:

$$\delta L_{\ell q} = \frac{1}{\Lambda^2} \sum_{ij} (\bar{e}_i \Gamma e_i)(\bar{q}_j \Gamma q_j) \quad \delta L_{qq} = \frac{1}{\Lambda^2} \sum_{ij} (\bar{q}_i \Gamma q_i)(\bar{q}_j \Gamma q_j), \quad (21)$$

where Γ represents a appropriate matrices. Here Λ is actually an energy denominator, associated with states of high mass. Because Λ is much larger than the initial state energy, this denominator is effectively constant. Such a term would produce a very different angular distribution than the standard model interaction, which always involves virtual states with one of the known vector bosons: the gluon, photon, W and Z. In Fig. 4, the effects of contact terms, interpreted as signals of quark compositeness, are compared to jet cross sections (see below) observed by the D0 detector at Fermilab[7]. The curves show the kind of limits that can be put on contact terms today, typically requiring the scale Λ to be a few TeV, roughly one thousand times the rest energy of the proton.

- Other favorite tests involve decays that are forbidden, or suppressed, in the standard model. Examples include a muon changing to an electron, and a bottom quark to a strange quark,

$$\mu \rightarrow e + \gamma, \qquad b \rightarrow s + \gamma, \qquad (22)$$

where the first is absent, and the latter rare, in the standard model. The existence of the former would imply physics "beyond" the standard model. The latter, which has been seen, requires a detailed evaluation of standard model predictions on rates and final states. Current limits[8] for $\mu \rightarrow e + \gamma$ are of the order of 10^{-11}; b decay to s occurs at the level of 10^{-4}.

- Symmetry violation. The archetype of symmetry violation searches are the famous parity-violation experiments of the fifties, which showed that the Hamiltonian of the weak interactions did not respect right/left reflection. Although this was a clue to the form of the relevant Hamiltonian, much more time, data and thought was necessary to discover the weak sector of the standard model. An illustration of how a symmetry violation manifests itself is shown in Fig. 5, the angular distribution of lepton
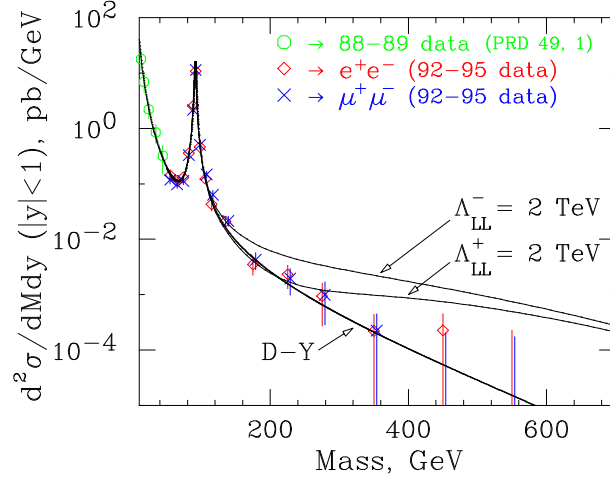
Figure 3: High mass lepton pairs at the Tevatron.

pairs in the decay of a Z at the SLD detector at SLAC, for different orientations of the spins of the incoming electrons that produced the Z.[9] If parity were respected in this experiment, the curves would be identical for different polarizations.

Experiments that search for new, heavy particles require imparting energies to conventional particles that are high enough to cross, or at least approach, the relevant energy thresholds. The menagerie of high energy machines consists of a few very large animals. Their species are: machines for electron-positron collisions, as at CERN (LEP II) and SLAC (SLC), electron-proton collisions, as at DESY (HERA), and proton-antiproton, or proton-proton collisions, as at Fermilab (Tevatron) and CERN (LHC). Many variations are possible, but the aim of searches is to find new physics through new particles and new effects in scattering final states. The search for rare decays and symmetry violation requies, in the main, rate rather than energy. Thus, for example, "factories" for the production of B mesons (CESR at Cornell, Belle at KEK and BaBar at SLAC) are tuned to an appropriate energy range where they are accumulating vast quantities of events.
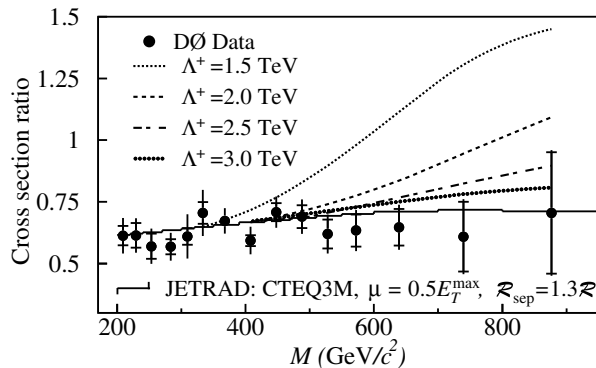
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Figure 4: Compositeness limits from the high mass jets at the Tevatron.

3.3 The Program: Ideal and Reality.

The ideal program for the determination of fundamental laws of nature in the language of quantum field theory is, of course, difficult to realize. On the experimental side, nature has evidently arranged energy scales in such a way as to make their exploration nothing if not challenging. Theoretical difficulties are in many ways analogous to those of Newtonian gravity. The very richness of any theoretical structure adequate to describe a varied class of phenomena requires new ideas and methods for its implementation. Thus, although expressions like Eq. (16) were derived very early after the invention of quantum field theory, it was a long time before anyone was able to do the sums over states in any but the simplest cases, and even today there are many restrictions to our abilities in this regard. In the remainder of this talk, I'd like to describe what some of these problems are, and indicate some of the methods developed to deal with them.

The most common, and fundamental, problem with the calculation of scattering amplitudes according to Eq. (16) is that the sums over intermediate states almost never converge without further input. In quantum field theory, the sums over free particle states are integrals over possible wave numbers. As it stands, we integrate over all wave numbers, including those that are arbitrarily large, corresponding to very high energy modes, and those that are very small, corresponding to low energy modes. By the usual correspondence of resolution to wavelength, the former are sensitive to the short-distance dynamics of the theory, the latter to its long-distance behavior.
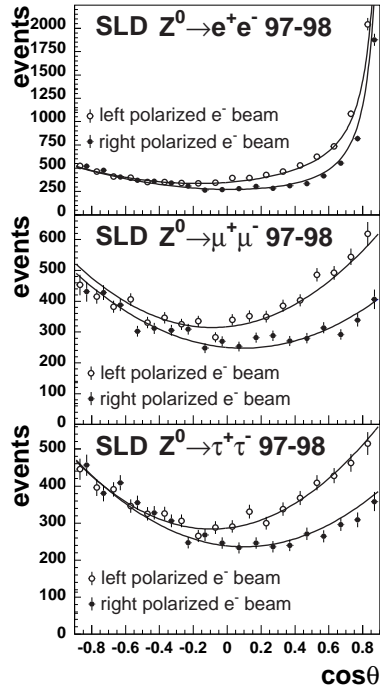
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Figure 5: Forward-backward distributions of lepton pairs. Symmetry under parity would require that the two curves be the same in each case.

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Problems arise at short distances because, if k is a typical wave number, transition amplitudes often increase with k, roughly as $V_{n,n-1} \sim k^a$, with $a = \sum (spins)$, where "spin" refers to the intrinsic angular momentum carried by each of the fields. This behavior leads to nonconvergence in most theories in four dimensions (even those with spinless particles). In particular, it limits the set of theories that have a reasonable perturbative interpretation at all.

Even when a theory can be controlled at short distances, integrals over long wavelengths can refuse to converge, a situation referred to as an infrared divergence. Infrared divergences indicate strong sensitivity to long-time behavior, and indeed, they are related to the difficulties encountered in the study of planetary motion in Newtonian gravity, which is sensitive to small, but cumulative, effects in the solar system.

Almost all scattering amplitudes suffer from either or both of these problems at some order in the potential, V. The question is therefore how to get any useful information at all out of such an ill-defined scheme! A partial, but workable, solution is to separate dynamics at different scales. That is, when the contributions from long or short wavelengths are not well predicted by our theory, we attempt to organize our ignorance into a few parameters, or even a few functions, that we take as given – determined by experiment. We reduce the theory to calculations over only those wave numbers that are not too large, and not too small. In the following section, we discuss how divergences at short distances are handled through renormalization, and at long distances, through infrared safety and factorization. The limitations of these approaches are many, including a general lack of mathematical definition, approximations that are at best asymptotic series, and corrections whose coefficients cannot be bounded rigorously. It is far from obvious at the outset that such a program can work, but in fact it has been remarkably successful.

4 Separating Scales

4.1 Short Times and Renormalization

Problems with the sum over states at short distances are illustrated by Fig. 6 in quantum electrodynamics. Here we show a low-order contribution to the development of a state with two electrons over some fixed time ΔT, during which a virtual intermediate state with an electron and a photon appears for a while. At the end, we revert to a two-electron state, but in the process momentum may be transferred, which we interpret as the quantum-mechanical origin of the force between electrons. In fact, the sum over states shown in the figure can be carried out in QED, and gives a result that has been known for a long time. We may decide, however, to take a closer look at the process,
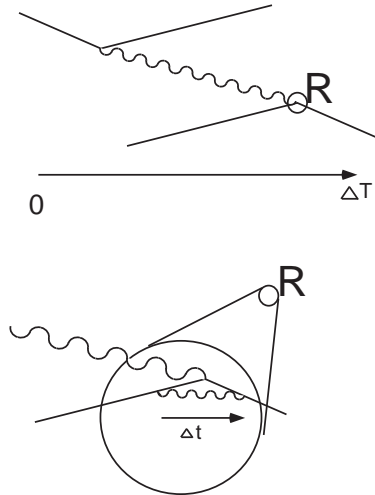
Figure 6: Gluon exchange and vertex correction.

in particular, at the region in time denoted by the circle R in the figure. The lower part of Fig. 6 shows a blowup of R, and on this scale we see that the absorption of the photon at the second vertex on top happened while the receiving electron was itself in a virtual state involving another photon. That state lasted for a time $\Delta t \ll \Delta T$, and we might well have missed it on the time-scale at the top of Fig. 6.

The practical problem in QED, QCD and any other four dimensional quantum field theory is that when $\Delta t \to 0$, all of these diagrams diverge, from their sums over very high energy states. By the correspondence between short times and high energies, these are the very short-lived states.

The solution to this problem, known as renormalization, is really rather straightforward. We simply replace all interactions like the bottom of Fig. 6, on time scales less than or equal to *any* fixed time T, by a known function, called the *running coupling* $g(\hbar/T)$, where $\hbar/T$ is the energy characteristic of frequency $1/T$. We would like to include in the running coupling the sum of the diagrams shown in Fig. 7, including *all* virtual states that are within time T, or distance cT, of the electron-photon vertex. Equivalently, we would like to sum over all intermediate states with energies greater that $\hbar/T$. Even though the sum does not converge, we can still identify the T-dependence of the running coupling, because an infinitesimal change in T only involves an
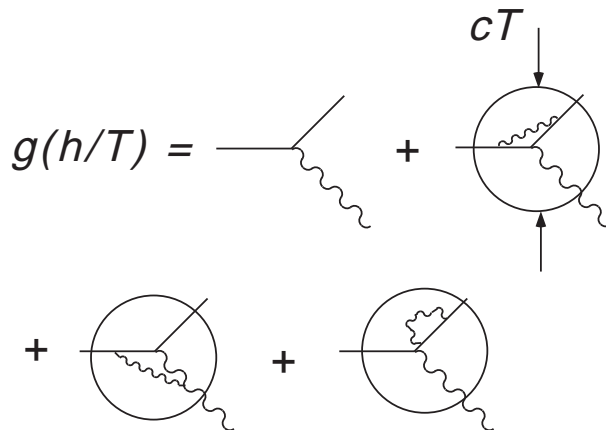
Figure 7: Lowest-order diagrams that contribute to the running coupling in QCD.

infinitesimal change in energy, and therefore depends on only a tiny part of the whole sum. Therefore, we can derive a differential equation for $g(\mu)$, of the form

$$\frac{\partial g(\mu)}{\partial \ln \mu} = \beta\left(g(\mu)\right) = -\frac{g^3(\mu)}{16\pi^2} b_0 + \mathcal{O}\left(g^5(\mu)\right) , \tag{23}$$

where a direct calculation from the diagrams of Fig. 7 gives

$$b_0 = 11 - 2n_f/3 \qquad \text{(QCD)} \tag{24}$$

with n_f the number of different relevant kinds (flavors) of quarks. Roughly speaking, a quark is relevant when its mass is no larger than μ, so that pairs of quarks may appear in virtual states on the time scale $T = \hbar/\mu$.

Once we have an approximate differential equation for the running coupling, we can solve for it, and we find,

$$g^2(\hbar/T) = \frac{16\pi^2}{b_0 \ln(\hbar^2/T^2\Lambda^2_{\text{QCD}}c^2)} , \tag{25}$$

where Λ_{QCD} is a mass that specifies a boundary condition – which can be taken as the value of $g(\mu)$ at any fixed value of μ. Λ_{QCD} is sometimes referred to as the "QCD scale", the scale at which the strong interactions become strong. Its actual value is not accessible to computation within QCD; presumably it is set

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by the physics at very short times, where the states are no longer adequately described by this theory. The magic of renormalization is that, whatever those states are, at low energies in QCD they express themselves only in the value of Λ_{QCD}. This *decoupling* of physics at very short times is what makes it possible for us to systematize our knowledge of QCD and the rest of the standard model self-consistently. It also helps us to understand why it is sometimes so difficult to see beyond the successes of the standard model.

4.2   The case of QCD

The solution, Eq. (25) for the QCD running coupling has features which are unique to that theory. In particular, when measured at short times, $T \rightarrow 0$, the coupling is weak, and even vanishes over very short time scales, or equivalently at very short wavelengths, or again equivalently, at very high energies. This property is called *asymptotic freedom*. Correspondingly, however, when measured over long times, or at low energies, the coupling becomes strong, and indeed diverges at times of order $1/\Lambda_{\text{QCD}}$. This behavior corresponds nicely to the paradoxical picture of QCD that emerges from high energy experiments. Over short distances, where large momenta can be transferred, the theory acts as though it were well described in terms of the states of $H_{\text{QCD}}^{(0)}$, that is, in terms of quarks and gluons. We shall see how shortly. But if we wait long enough (or go far enough into the past), the quarks and gluons *always* and without fail conspire to form color-neutral states $|m_{final}\rangle$ ($|m_{initial}\rangle$), consisting of mesons and baryons with $q\bar{q}$ and qqq quantum numbers, occasionally with quark-less "glueballs". This is known as *confinement*. Qualitatively, all this is consistent with the behavior of the running coupling in (25), but it is not immediately obvious how to combine these features into a usable theory. Here again we turn to a separation of scales. In this case, we shall learn how to separate long times from short.

4.3   Large Times and Physical Pictures

To separate long from short times in matrix elements, we need to see how the former contribute to time-ordered perturbation theory. Certain very general features are easy of identify. For this purpose, we return to Eq. (15), and see that the time integrals are all of the same kind,

$$\int_{-\infty}^{t} d\tau_n \, e^{-i(S_{n-1}-S_n)\tau_n} \int_{-\infty}^{\tau_n} d\tau_{n-1} e^{-i(S_{n-2}-S_{n-1})\tau_{n-1}} \dots$$
$$\times \int_{-\infty}^{\tau_2} d\tau_1 e^{-i(S_0-S_1)\tau_1} \,. \tag{26}$$

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In fact, it is not so easy to generate sensitivity to long-time intervals in these expressions, because all the time dependence is in exponentials, and oscillating t-dependence suppresses large times as the integrals cancel over each oscillation. The exception to this principle is at points of stationary phase in Eq. (26). Examining the exponents shows that the total phase has a curious interpretation, which may be represented as:

$$
\begin{aligned}
\text{PHASE} &= \sum_{\text{states } m=1}^{n} S_m(\tau_m - \tau_{m-1}) \\
&= \sum_{\text{states } m=1}^{n} \left(\sum_{\text{particle } j \text{ in } m} \omega(p_j) \right) (\tau_m - \tau_{m-1}) \\
&= \text{FREE} - \text{PARTICLE ACTION}.
\end{aligned}
\tag{27}
$$

Thus, the phase may be thought of as the sum of all the *classical* action accumulated in the intermediate states, consisting of free particles. In other words, stationary phase corresponds to stationary classical action. But stationary action corresponds to physical motion in the classical mechanics of particles. Thus, surprisingly, sensitivity to long times requires that the succession of states in the computation of the scattering amplitude have a description as a succession of free particles propagating between points in space-time. When this is *not* the case, the amplitude is dominated by canceling phases, and there is no sensitivity to long-time dynamics. Quantities that are independent of long-time dynamics are sometimes called *infrared safe*.

4.4 Infrared Safe Cross Sections

Are there any infrared safe observables? By definition, they must be independent of how quarks are confined, or any other long-time properties of QCD. In fact, there are many, but they always involve *inclusive* measurements. They are seen most directly in e^+e^- collisions, as at the LEP machine at CERN. The simplest example is probably the total cross section for an electron-positron pair to annihilate, that is to mix with a state which consists of a single photon, which subsequently decays to hadrons, $\sigma_{\text{tot}}^{(e^+e^-)}$. [e] Alternately, the electron-positron pair may be just energetic enough to produce an on-shell Z particle,

---

[e]The photon state through which the system passes is usually referred to as an "off-shell" photon, because its energy E is much larger than pc, its momentum times the speed of light. $E = pc$ is the "mass shell" relation for the (massless) photon.

Figure 8: The optical theorem for Z decay.

whose decay rate, Γ_Z, can then be measured. The rate is just a normalized probability per unit time (the larger the probability the shorter the lifetime).

The first step in showing that Γ_Z is infrared safe is to invoke the conservation of probability, in the form of the optical theorem of quantum mechanics, which states that the total decay probability per unit time is (up to some constants which we ignore) the imaginary part of the amplitude for *forward* scattering. This is shown schematically in Fig. 8, whose left-hand side represents the sum over all possible final states for the decay of a Z particle (the dashed line). By the optical theorem, then,

$$\Gamma_Z \sim \text{Im}\,\Pi_Z(Q^2 = m_Z^2)\,, \tag{28}$$

where $\Pi_Z(Q^2)$ is the imaginary part of the forward-scattering amplitude, for a Z to change into a Z by passing through any and all intermediate states that the field theory allows.

Now let's think about the forward scattering amplitude. We ask whether there are any physical pictures for $\Pi_Z(Q^2)$, in which the Z decays at a point into particles which travel freely, rearrange themselves at points into other states, but eventually come back together to form the Z. There are no such physical pictures, because after the Z decays the particles it produces will be moving rapidly in opposite directions, as illustrated in Fig. 9, and there is no way for them to reassemble the Z by free propagation. Like Humpty-dumpty, the Z cannot be put back together again: q.e.d.

This may seem a bit abstract, but if we go ahead and calculate the Z decay rate using Eq. (16), the infrared safety that we have just proved guarantees that we will find a completely finite answer. That is, the sums over intermediate
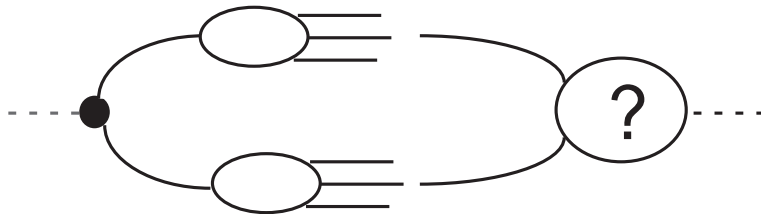
Figure 9: Lack of physical processes for Z decay

states converge at long wavelengths, and we find a finite expansion in $\alpha_s(M_{\mathrm{Z}})$:

$$\Gamma_{\mathrm{Z}} = \Gamma_{\mathrm{Z}}^{(\mathrm{EW})} \left(1 + \sum_{n=1}^{\infty} c_n \alpha_s^n(M_{\mathrm{Z}}) \right) , \tag{29}$$

where $\Gamma_{\mathrm{Z}}^{(\mathrm{EW})}$ is the decay rate with no QCD interactions at all (EW stands for electroweak). In the expansion, $\alpha_s \equiv g^2/4\pi$ is the QCD "fine-structure constant". Because all the integrals are finite, we can choose the scale of our running coupling to be M_{Z}. Then, neglecting the masses of the quarks, the coefficients c_n are just numbers, because there are no further dimensionless ratios on which they can depend. For example, $c_1 = 1/\pi$.

With one more technical observation, we can extend these considerations to a much larger class of observables: the *jet* cross sections. A jet in $\mathrm{e^+e^-}$ annihilation or Z decay is just a collection of particles moving in more-or-less the same direction. The complete details of the identities and momentum spectra of particles within a jet do not turn out to be infrared safe, but the probability distribution for a jet with a specified *total* energy moving in a specified direction, does turn out to be infrared safe.

Suppose we ask for the probability, not just that the Z should decay, but that it should decay in such a way that all but a small fraction, ε, of its energy flows into two oppositely-directed cones of angular size δ:

$$\sigma_{2\mathrm{J}} = \sigma(E_{\mathrm{cones}\,(\delta)} \geq (1 - \varepsilon)m_{\mathrm{Z}}) . \tag{30}$$

This would be a very unlikely arrangement if particles were distributed randomly in the final state. Imagine, however, a Hamiltonian for QCD in which particles are produced *only* in the directions of these cones. Such a Hamiltonian would not be Lorentz, or even rotationally invariant, but it could be a bona-fide Hamiltonian anyway, and we could apply all the reasoning that we

used for the full Hamiltonian to this truncated one. If we use unitarity for such a Hamiltonian, the sum over all of its available states will be infrared safe, just as for the full Hamiltonian, and should be computable in perturbation theory, with a result of the form

$$\sigma_{2J} = \sigma_0 \sum_n c_n(\delta)\, \alpha_s^n\,, \tag{31}$$

with σ_0 a convenient normalization and with coefficients c_n that depend on the opening angles δ. The crucial observation now is that the theory with this truncated Hamiltonian has, for this particular cross section, exactly the same set of points of stationary phase in the sum over its intermediate states, and therefore the same long time behavior, as the *full* theory. This is because for a jet final state the physical pictures of evolution in the full theory *only* involve the decay and recombination of particles within the jet cones, or the emission of very soft particles. Intermediate states with energetic particles moving in directions outside of the jet cones are never at points of stationary phase, because once outside the cones, the particles never encounter other particles that could scatter them back in. Thus, the truncated and full theories only differ by short-distance, infrared safe, corrections in perturbation theory. As a result, the form (31) holds for full QCD perturbation theory. This reasoning can be extended to a large class of jet-related cross sections in e^+e^- annihilation. To be specific, we can identify a flexible criterion: any cross section that is insensitive to collinear rearrangements and to emission of soft gluons is infrared safe.

If we measure a cross section for jets defined in this fashion, we have a way of "seeing" quarks and gluons. Of course, there is no unique jet definition; each event contains, according to quantum mechanics, a sum over its possible histories. For infrared safe cross sections, however, we can compute corrections to the dominant history, and predict the outcomes of variant definitions of jet events. In this way, we can design experiments to study the true short-distance behavior of a theory, even though at short times the very degrees of freedom are different from those that we detect directly.

4.5 Factorization and Deep-inelastic Scattering

With jet cross sections, we have opened a window to the short distances of quantum chromodynamics. The set of such infrared safe cross sections is rather limited, however, and does not include any experiment where we start out with strongly interacting particles (hadrons). Why not? When there are hadrons in the initial state, the strong interactions have been going on since time immemorial; the quarks in a random proton may well have been interacting with

each other since the big bang. Thus, there is no hope that these long-time interactions cancel out. They were there to begin with, and their presence actually defines the cross section. Still, we need not abandon hope. Since every proton is the same, initial-state interactions may appear as overall factors in suitably-defined cross sections. The trick is to identify the appropriate class of reactions. In a sense, this is easy. We should study the same sets of final states as in e^+e^- annihilation – inclusive cross sections, but always involving the large momentum transfers that imply short-time interactions. The archetype of these reactions is deep-inelastic scattering, from which, in fact, arose the first direct evidence for quarks.

The deep-inelastic scattering of an electron and a proton (or nuclear) target involves reactions of the sort

$$e(k) + N(p) \rightarrow e(k') + X(p+q)\,, \tag{32}$$

where the invariant momentum transfer, $q^2 \equiv -Q^2 = -(k'-k)^2$, and energy transfer in the target rest frame, $\nu \equiv (p \cdot q)/m_N$ are both large. The first condition ensures that the scattering takes place over a short distance, the second that it takes place over a short time. In this case, the process proceeds at the quantum mechanical level through the exchange of a photon between the electron and a quark in the target, similarly to the top of Fig. 6.

Because the electromagnetic interaction is much weaker than the strong, most of the complexity is in the succession of states of the proton, which fluctuate much more rapidly than those of the electron. As a result, we can work to "lowest order" in electromagnetism, and keep only states in which a single photon is exchanged.

In a typical succession of states that describes this process in field theory, the electron and proton fluctuate into an electron plus a photon for the former, and a collection of quarks and gluons for the latter. In this, rather complex, intermediate state, one of the quarks absorbs the photon, to produce a state which is almost the same as the original virtual state of the proton, differing only in the momentum of the "active" quark, the one that received the photon, which is now moving in a new direction, with an energy that is much larger than the rest mass of the proton. The basic reaction is the same as in the transition from state m_0 to state m_1 in Fig. 1b.

The experiment is particularly simple, because to measure the momentum transfer, q, it is only necessary to look at the electron in the final state. The inclusive cross section for all deep-inelastic reactions with the same momentum and energy transfer, has much in common with the total electron-positron annihilation cross section, and can be treated in much the same way. Summing over all final states by means of the optical theorem, we again arrive

at a forward-scattering cross section. Now, instead of a decay, however, the forward scattering describes a process in which the intermediate photon is absorbed by the constituents of the proton, and is then reemitted, into the same momentum state. Just as in the annihilation case, the absorption and reemission cannot be separated by states in which the scattered, active quark propagates too far. If it did, it would recede too far from the remnants of the proton for the proton to reform in the forward scattering. The only physical pictures available for forward scattering are those in which the photon is absorbed and emitted at points that are separated by a light-like distance within the proton. In the center-of-mass system of the proton and virtual photon, Lorentz contraction makes even this is a short distance. As a result, all of the effects associated with the evolution of final states, such as the probabilities of producing jets, are calculable, just as in electron-positron annihilation. The same is not true of interactions in the initial state. These are, however, quite independent of the deep-inelastic scattering itself. These arguments suggest that the short-distance interactions, including those that determine the jet structure of the final state, are quantum-mechanically incoherent with the initial state interactions that bind the hadronic target. This incoherence has powerful implications.

First of all, because the proton target has been around for a long time, the contribution of the initial, struck quark, $\mathcal{Q}$, to the energy deficits of states before the interaction must be relatively small. For this to be the case, the momentum of that quark should be given approximately by ξp, with $0 \leq \xi \leq 1$. A quark with a large transverse momentum, or with a fraction ξ outside this range, would automatically produce a state that is very short-lived, and which would therefore contribute a perturbatively-calculable correction. At the same time, the incoherence of the scattering process with the binding of the proton means that the scattering can be described by transitions between an initial state, $\gamma + \mathcal{Q}$, to a set of final states, the simplest of which is a single quark $\mathcal{Q}$,

$$
\begin{aligned}
\gamma(q) + \mathcal{Q}(\xi p) \rightarrow\ & \mathcal{Q}(\xi p + q)\,, \\
\rightarrow\ & \mathcal{Q}(\xi p + q - k) + \mathcal{G}(k)\ \dots\,,
\end{aligned} \tag{33}
$$

with $\mathcal{G}$ a gluon. The requirement that the final-state jets produced by the struck quark be physical is $(\xi p + q)^2 > 0$, which, neglecting masses, leads to the restriction

$$
\xi \geq \frac{Q^2}{2p \cdot q} \equiv x\,, \tag{34}
$$

where x is called the "Bjorken scaling variable".

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In technical terms, these considerations are summarized by a set of very general relations. For simplicity, let's stick to the fully inclusive cross section, which depends only on the momentum transfer q and the momenta of the initial particles, k for the electron, and p for the target nucleon. The "one-photon" approximation for the electrodynamics part of the virtual states allows us to write the cross section as a product of "leptonic" and "hadronic" functions:

$$\sigma_{eN} = L_e(k, k') \ \times \ W_N(q, p) \, . \tag{35}$$

The incoherence between initial and final state strong interactions in the scattering process is expressed in what is known as a factorization formula for the hadronic part,

$$W_N(q, p) = \sum_{a=\mathcal{Q}, \bar{\mathcal{Q}}, \mathcal{G}} \int_x^1 dy \ C_a \left(\frac{q^2}{\mu^2}, \frac{x}{y}, \alpha_s(\mu) \right) \ f_{a/N}(y, \mu) \, . \tag{36}$$

The sum is over all parton types. At higher orders in QCD, even a gluon may scatter from the photon, by transforming itself into a virtual state with a quark-antiquark pair. The functions C_a are called coefficient functions. They describe the dynamics of the possible scatterings in Eq. (33) at short times, and are calculable in perturbation theory. The $f_{a/N}$ are parton distribution functions, which interpolate between hadrons and partons. The scale μ in these functions is the inverse of the largest lifetime of a state that we allow into C_a: $\mu = 1/T_{\max}$. Quantum mechanical incoherence implies that these functions appear as a factorized product in the calculation of W_N. At present, we can't calculate the parton distributions, but we can measure them, by fitting Eq. (36) to data over a range of momentum transfers, and by observing weak boson as well as photon exchange.

The parton distribution functions may be expressed as expectation values in states of the target, for example, for quark $\mathcal{Q}$ in nucleon N,

$$f_{\mathcal{Q}/N}(\xi, \mu) = \int d\lambda \ e^{-i\lambda \xi p \cdot n} \ \langle N(p) | \bar{\mathcal{Q}}(\lambda n) \Gamma_{\mathcal{Q}} \mathcal{Q}(0) | N(p) \rangle \, , \tag{37}$$

where $\mathcal{Q}$ is the corresponding quark field operator, and where $\Gamma_{\mathcal{Q}}$ is a matrix that projects out a number operator for quarks of momentum fraction ξ. The vector n^μ represents the light-like velocity of the struck quark. The factorization scale μ enters this expression as the scale of renormalization in the perturbative calculation of the matrix element.

A striking consequence of asymptotic freedom follows from Eq. (36). As the momentum transfer Q increases, we may take the factorization scale μ to

be large, of order Q. But at any x and Q, W_N must be independent of our choice of μ:

$$\mu \frac{dW_N(p,q)}{d\mu} = 0 \,. \tag{38}$$

This means that the μ-dependence of the parton distributions must be compensated by that of the coefficient functions. But the coefficient functions are calculable in perturbation theory. Hence, so must be the μ-dependence of the parton distributions:

$$\mu \frac{df_{a/N}(\xi,\mu^2)}{d\mu} = \sum_{b=\mathcal{Q},\bar{\mathcal{Q}},\mathcal{G}} \int_\xi^1 dz \, P_{ab}(\xi/z, \alpha_s(\mu)) \, f_{b/N}(z,\mu) \,. \tag{39}$$

The only dimensional scale upon which the "evolution kernels" P_{ab} may depend is μ in $\alpha_s(\mu)$, because μ is the only such variable held in common by C and f. This means we can take parton distributions determined from one set of data, and apply them to predict scattering at much higher, or lower, momentum transfers, to any scale for which the running coupling remains small.

As the factorization scale μ increases, the coupling becomes weaker and weaker, and the importance of extra gluons in the final state in Eq. (33) becomes less and less, until only the simple *quark + photon $\rightarrow$ quark* process remains. Eventually, W_N becomes barely dependent on Q, a property known as *scaling*, in which dependence on the scaling variable x of Eq. (34) is all that remains. Scaling was for QCD what elliptic orbits were for Newtonian gravity, a dramatic explanation of a striking, but previously unaccountable, observation.

Beyond this, the factorization formalism makes it possible to predict cross sections for jets and particle production at arbitrary length scales, not only in electron-nucleon scattering, but also in nucleon-nucleon scattering. As a practical matter, this enables us to make predictions for higher energies, and to search for new physics, perhaps some new particle F of mass Q. In hadronic collisions, the factorized inclusive cross section for any such "F-production", at center-of-mass energy E_{cm}, involves two momentum fractions, one for each hadron,

$$Q^4 \frac{d\sigma_{AB \rightarrow F(Q)+X}}{dQ^2} = \sum_{a,b} \int_0^1 \frac{d\xi}{\xi} \int_0^1 \frac{d\eta}{\eta} \, \hat{\sigma}_{ab \rightarrow F(Q)+X}^{(\mathrm{PT})} \left(\frac{Q^2}{\xi \eta E_{\mathrm{cm}}^2}, \mu, \alpha_s(\mu) \right)$$
$$\times f_{a/A}(\xi,\mu) \, f_{b/B}(\eta,\mu) \,, \tag{40}$$

with the same parton distributions f as in deep-inelastic scattering. As for C_a in Eq. (36), $\hat{\sigma}_{ab}^{(\mathrm{PT})}$ is perturbatively calculable. It is on the basis of formulas

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such as this, that the Tevatron, and subsequently the LHC, will search for new physics, by constantly comparing data to predictions based on the standard model and its extensions, through a cycle of tests as described above. Already volumes of calculations and predictions exist, each implementing a proposal for what will be found in the short-distance cross sections $\hat{\sigma}_{ab}^{(\mathrm{PT})}$. The curves in Figs. 3 and 4 were calculated in this way. Nature will decide which, if any, matches the data.

5   Conclusions

In broad terms, the physics of fundamental processes today continues as it did three centuries ago, though tests of our existing understanding of matter and forces. These tests have a dual role, first in the elaboration of our current models, and second, in a search for their limitations.

We are far from a full command of our present quantum field theories. Our ability to separate scales, however, enables us to probe different sectors of a theory on paper, on the computer screen and in the laboratory, through renormalization and factorization. For QCD, in particular, each hadronic event displays the imprint of dynamics at all scales. In the observation of its dynamics we cannot escape the dualities of weak and strong, fundamental and effective. The mathematician-physicists of the eighteenth century developed tools to apply the law of gravitation to the motions of the moon and planets, from the book of astronomical observation. So are we attempting to apply modern field theory to hadronic scattering, and to read the quantum mechanical history of chromodynamics in the alphabet of its final states.

The search for new physics, to reveal a structure that will explain the standard model and its apparent complexities in terms of something (hopefully) simpler, may only require the energy necessary to narrow an energy deficit, give a heavy virtual state just a little longer lifetime, and produce the kinds of signals described in Sec. 3.2. Because of the reach of hadronic colliders, in particular the Tevatron and the LHC, these will most likely be the key to unlock the next energy scale. If they do, a new chapter in our understanding of fundamental process will begin, with a new world of physics and mathematics to explore.

Acknowledgements

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